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A NOTE ON THE MEASUREMENT OF K AND J UNDER SMALL SCALE YIELDING CONDITIONS USING THE METHOD OF CAUSTICS

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Experimental techniques for the direct optical measurement of the value of the J-integral allow the determination of J for planar cracked bodies of arbitrary geometry and loading. Several methods such as photoelastic coatings [1], moire interferometry [2], and caustics [3] have been successfully applied to the measurement of J in elastic-plastic materials under static loading conditions. In this report, preliminary experimental investigations of the applicability of caustics to elastic-plastic fracture are communicated. All the experiments are performed under conditions of small scale yielding.

Consider a parallel beam of light incident on an initially optically flat, polished planar fracture specimen. The out of plane near crack tip deformation of the specimen, due to loading, causes the reflected rays to deviate from parallelism. If certain geometrical conditions are met, a shadow spot bounded by a bright curve (caustic), is formed on the film plane of a camera placed in front of the specimen to collect the reflected light. To obtain an image of the shadow spot the camera is focused not *on* the specimen but *behind* it at a distance z_0. For further details of the method of caustics see [4] and [5].

The light that forms the caustic curve is reflected from a unique curve on the specimen called the *initial curve*. Let the distance from the crack tip to the initial curve along $x_2 = 0$, be called the initial curve radius and denote this distance by r_0, where x_2 is the abcissa of a local coordinate system centered at the crack tip as shown in Fig. 1. Since the caustic curve comes only from light reflected from the initial curve, whatever data are obtained come from essentially just one curve. By varying z_0, the initial curve radius r_0 may be varied and thus data may be taken from varying distances from the crack tip.

Under conditions of small scale yielding the elastic $1/\sqrt{r}$ singular field dominates at some distance outside the plastic zone. Thus for $r_0 >> r_p$, where r_p is the plastic zone size, the analysis of caustics based on the elastic crack tip fields [5] may be used yielding

$$K_I = ED^{5/2}/10.7z_0\nu t \tag{1}$$

and

$$r_0 = 0.316D \tag{2}$$

where K_I is the mode I stress intensity factor, D is the maximum diameter of the caustic in the x_2 direction, E is the modulus of elasticity, ν is Poisson's ration, t is the specimen thickness, and z_0 is the distance from the specimen to the plane the camera is focused on.

Equation (1) is valid only when the initial curve is outside the plastic zone ($r_0/r_p >> 1$). To quantify how large r_0/r_p must be, compact tension specimens of annealed 4340 steel and of cold rolled 1018 steel were tested. Specimens 29, 30, 30A, and 44 illustrated in Fig. 1 were used in these tests. The material properties of the 4340 steel are $\sigma_0 = 827$ MPa, $\alpha = 3.3$, and $n = 8.7$ for a best fit to the Ramberg-Osgood material model. The 1018 steel had $\sigma_0 = 560$ MPa and no hardening, ($n \to \infty$).

The experiment consisted of determining K_I from caustics (K_{caus}) and from boundary conditions (K_{BC}) while varying the ratio of the initial curve to plastic zone size. The initial curve radius is determined from (2). The results are given in Fig. 2 where K_{caus}/K_{BC} is plotted as a function of r_0/r_p. The ratio r_p/w in the figure, where w is the uncracked ligament length, indicates the extent of the plastic zone relative to the specimen dimensions. According to earlier results presented in [6] plane stress conditions prevail at a distance $r \geq 0.5t$, where t is the specimen thickness. Therefore only data from caustics with $r_o/t > 0.5$ are plotted, thus insuring plane stress conditions. Figure 2 demonstrates that for the materials tested r_0/r_p must be greater than 1.5 for the elastic caustics analysis to hold in small scale yielding. This ratio is consistent with the value of 2 given in [7] for an analysis based on a plane stress Dugdale model. Further details and results of these experiments will be discussed in a planned full paper.

In similar experiments, caustics formed due to reflection of light from within the crack tip plastic zone ($r_0/r_p < 1$) were investigated with the goal of determining how to use caustics to measure the J integral directly. The same 4340 and 1018 steels as before were used. The specimens used were 3, 28, 30A, 42, and 44 illustrated in Fig. 1.

The existing analysis of caustics for plastically deforming crack tip regions is valid only when the initial curve lies well within the plastic zone where the plane stress HRR fields dominate. When this condition is satisfied the numerically predicted caustic shape is as shown in Fig. 3a, for hardening exponent $n = 9$. Figure 3b shows a caustic found experimentally for $n = 8.7$. The agreement between the two figures is very good. For this particular case $r_0/r_p = 0.3$.

When the initial curve is within the plane stress, HRR dominant region, J for a hardening material is related to the caustic diameter by [8]

$$J = S_n \alpha \varepsilon_0 \sigma_0 [1/\alpha \varepsilon_0 z_0 t]^{(n+1)/n} D^{(3n+2)/n} \qquad (3)$$

where ε_0 is the yield strain and S_n is a numerical factor dependent on n and given in [8]. For a non-hardening material [3]

$$J = \sigma_0 D^3/13.5 z_0 t \qquad (4)$$

and

$$r_0 = 0.363D \qquad (5)$$

For a material of low hardening r_0 is well approximated by (5). The actual near tip crack field is three dimensional but approaches the plane stress field as r/t increases. The three dimensionality of the field was investigated by experimentally determining J_{caus}/J_{BC} as a function of r_0/t, using (3) or (4) to calculated J_{caus} and (5) to calculate r_0. The data in Fig. 4 show this relationship for small scale yielding. Only data from caustics with shapes similar to the predicted HRR shape are presented. Thus the data show only the effect of three dimensionality. J_{caus}/J_{BC} approaches unity at around $r_0/t = 0.6$ or 0.7. This indicates that r_0/t must be greater than 0.6 or 0.7 for (3) and (4) to be valid. Further tests with the purpose of more thoroughly investigating the three dimensional effects are under way.

A sequence of caustics obtained by reflection of light from different distances from the crack tip is shown in Figs. 5a-5f. These photographs were obtained by varying r_0/r_p while keeping the load applied to the specimen constant. For $r_0/r_p > 1$ (Fig. 5f) the caustic retains the shape predicted by the analysis of caustics based on the elastic crack tip fields [5]. As r_0/r_p is reduced, the shape deviates from the elastic shape, reaching the shape predicted by the analysis of caustics based on the HRR crack tip field [8] when $r_0/r_p \leq 0.37$ (Figs. 5a and 5b). As explained earlier, only caustics with shapes corresponding to either elastic or HRR field predictions were analyzed here. Caustics in the transition region between the two limits cannot be analyzed since a full-field, analytical, elastic-plastic solution for the crack tip fields is not as yet available.

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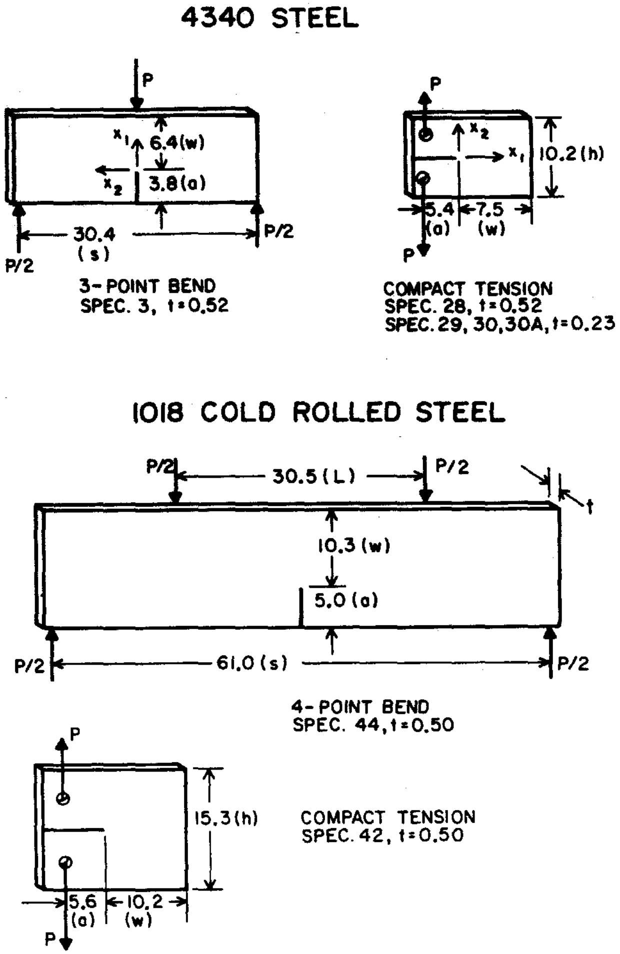


Figure 1. Specimen geometries. All dimensions in cm.

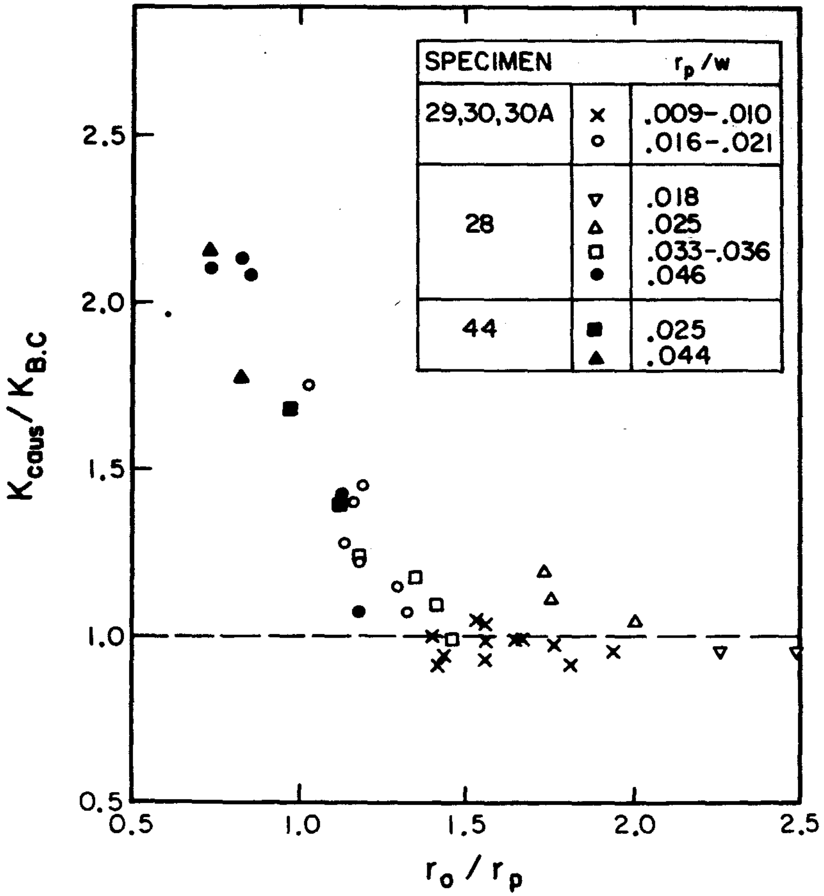
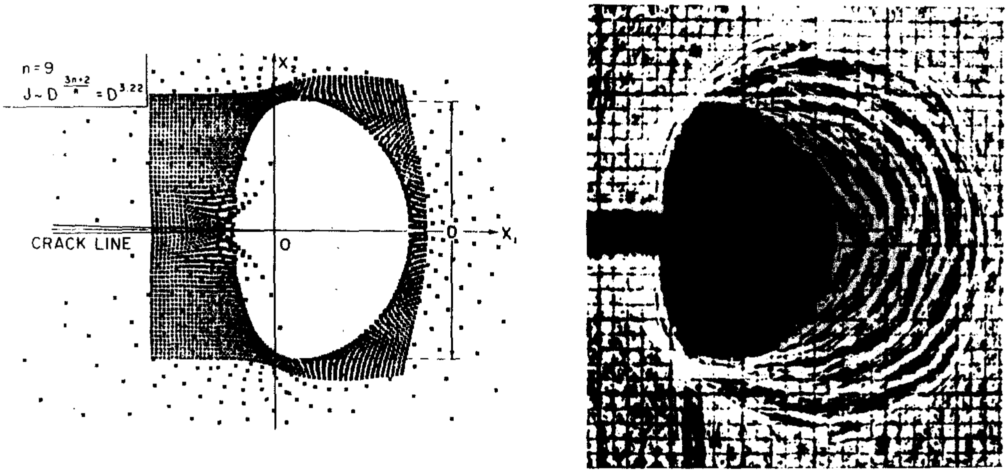


Figure 2. K_{caus}/K_{BC} s. r_0/r_p. Deviation from unity shows error caused by plasticity when applying caustics to elastically deforming regions that are just outside the plastic zone.



(a) (b)

Figure 3. Caustics formed due to reflection of light from well within crack tip plastic zone. (a) Numerically simulated (from [8]) (b) Experimental.

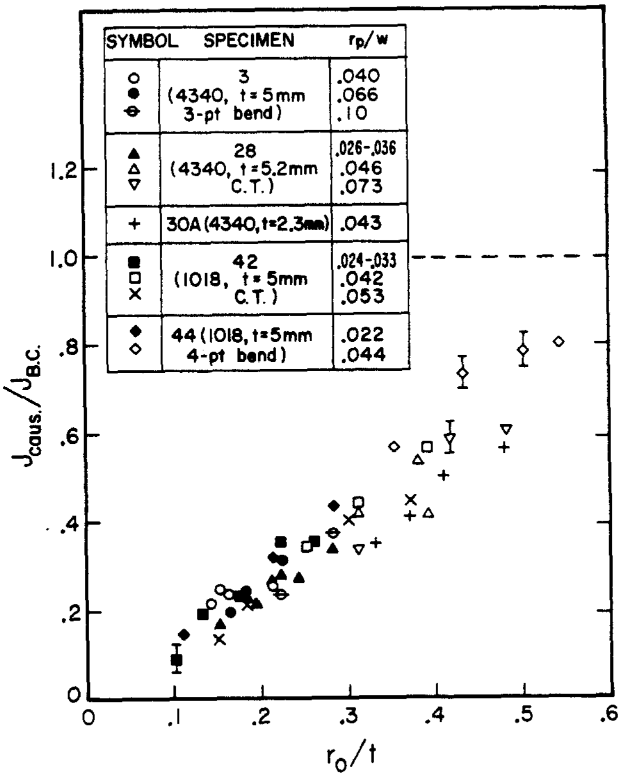
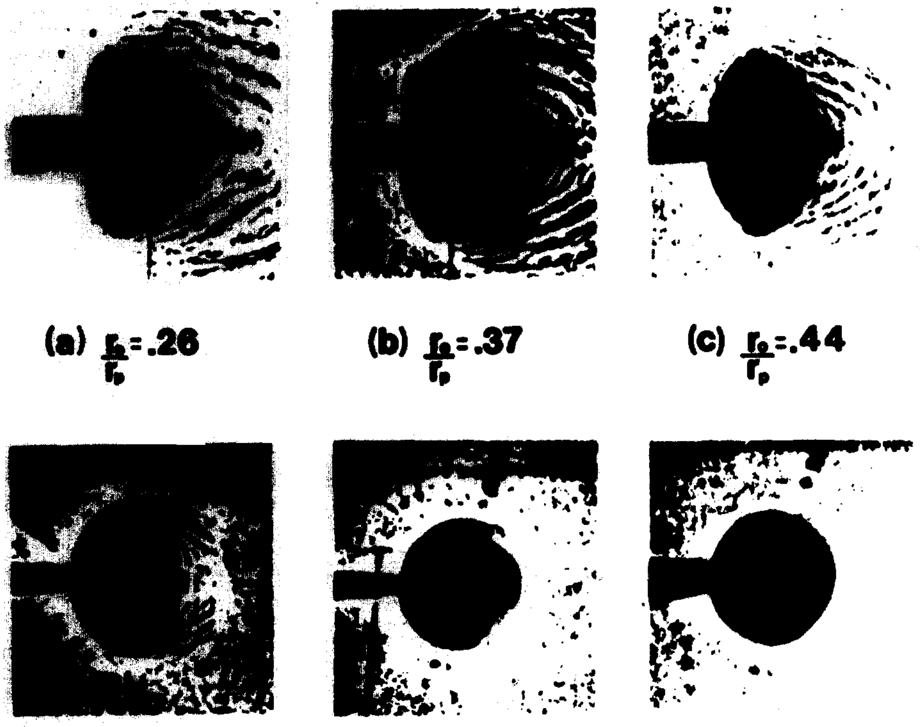


Figure 4. J_{caus}/J_{BC} vs. r_0/t. Deviation from unity indicates three dimensional field for r/t < 0.6 or 0.7.



(a) $\frac{r_o}{r_p}=.26$ (b) $\frac{r_o}{r_p}=.37$ (c) $\frac{r_o}{r_p}=.44$

(d) $\frac{r_o}{r_p}=.60$ (e) $\frac{r_o}{r_p}=.70$ (f) $\frac{r_o}{r_p}=1.1$

Figure 5. Sequence of caustics obtained for reflection of light from regions near a plastically deforming crack tip.